

# Graph Theory: Homework Set 1

October 8, 2008

1. Show that in any graph the number of vertices of odd degree is even.
2. Prove that every graph contains two vertices of equal degree.
3. Find a graph with 5 vertices and exactly 22 cycles.
4. Let  $G$  be a graph with  $n$  vertices. Prove that if  $e(G) > \binom{n-1}{2}$  then  $G$  is connected.
5. Show that any graph  $G$  has at least  $\binom{\chi(G)}{2}$  edges.
6. For each  $k \geq 3$ , find a bipartite (i.e. 2-colourable) graph  $G_k$ , and an ordering  $v_1, \dots, v_{n(k)}$  of its vertices, such that the greedy algorithm uses  $k$  colours to colour  $G_k$  (when its vertices are coloured in the order  $v_1, \dots, v_{n(k)}$ ).
7. Given a graph  $G$ , order its vertices in such a way that the greedy algorithm uses only  $\chi(G)$  colours to colour  $G$ .
8. How many ways are there of seating  $n$  ladies and  $n$  gentlemen around a circular table with  $2n$  seats, in such a way that ladies and gentlemen alternate? Reformulate this as a problem in graph theory (and solve it).
- 9\*. How many ways are there of seating  $n$  married couples around a circular table with  $2n$  seats, in such a way that nobody sits next to their spouse? (**Hint.** Use the inclusion-exclusion formula.)
- 10\*. How many ways are there of seating  $n$  married couples around a circular table with  $2n$  seats, in such a way that ladies and gentlemen alternate *and* nobody sits next to their spouse? (This is the famous *problème des ménages*.)