Graph Theory: Homework Set 2

October 24, 2008

1. Determine the edge chromatic number of $K_{m,n}$.
2. Prove that a regular graph of degree 5 cannot be decomposed into subgraphs, each isomorphic to a path of length 6.
3. Let $\bar{G}$ denote the complement of G (on the same vertex set as G). Show that

$$\chi(G) + \chi(\bar{G}) \geq 2\sqrt{n}.$$

4. Show that a graph of order n and size $(k-1)n - \binom{k}{2} + 1$ contains every tree of order $k+1$.
5. Show that a graph with n vertices and minimum degree $\lfloor \frac{(r-2)n}{r-1} \rfloor + 1$ contains a K_r.
6. Let $\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n} \in \mathbb{R}^d$ for some d. Suppose that $||\mathbf{x_i}|| = 1$ for all i. Prove that there are at most $\lfloor n^2/4 \rfloor$ unordered pairs i, j such that $||\mathbf{x_i} + \mathbf{x_j}|| < 1$.
[**Hint.** Show that $||\mathbf{x_i} + \mathbf{x_j}|| \geq 1$ for some $1 \leq i < j \leq 3$.]
7. Prove that the Ramsey number $R(3,4)$ is 9.
8*. By considering the graph on the integers modulo 17 in which i is joined to j iff $i - j$ is a square mod 17 (i.e. $1, 2, 4, 8, 9, 13, 15$ or 16), prove that $R(4,4) = 18$.