

# Graph Theory: Homework Set 3

November 13, 2008

1. Show that every graph with at most three cycles is planar.
2. Find three non-isomorphic (i.e. different) 5-regular planar graphs.
3. Prove that there is only one 5-regular maximal planar graph.
4. Find two graphs with degree sequence  $(6, 5, 5, 5, 3, 3, 3)$ , one planar and one non-planar.
5. Find a planar graph with 8 edges that has no plane drawing in which every finite region is convex.
6. Prove that the number of trees with  $n - 1 \geq 2$  labelled *edges* is  $n^{n-3}$ .
7. Let  $n = 2^p$ . Show that  $K^{n+1}$  is not the union of  $p$  bipartite graphs but that  $K^n$  is. Deduce that if there are  $2^p + 1$  points in the plane then some three of them determine an angle of size at least  $\pi(1 - 1/p)$ .
8. Complete the proof of Wagner's Theorem given in the book by showing that every pentagon (with no crossing edges) is star-shaped. Is the same true for hexagons?